Masters Thesis Information Showcase
Spring Semester-2026
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Department: Mathematics
Programme: Masters of Science
Specialization: Applied Mathematics

1	**Title of the Thesis** **Mathematics Behind Fuzzy Graph Neural Networks for Analyzing Real-World Datasets**
2	**Abstract** (min 300 to 500 words) Graph Neural Networks (GNNs) have emerged as powerful deep learning techniques for analyzing graph-structured data by learning complex relationships among interconnected entities. However, conventional GNN and Social Network Analysis (SNA) approaches may inadequately represent uncertainty, ambiguity, and varying strengths of relationships commonly observed in real-world networks. This study investigates the mathematical foundations and practical implementation of Fuzzy Graph Neural Networks (FGNNs) by integrating fuzzy logic, graph theory, social network analysis, and graph deep learning. The proposed framework aims to enhance the representation and analysis of uncertain relationships through fuzzy membership values rather than relying exclusively on binary connections. The methodology evaluates traditional centrality measures and community-detection approaches alongside GNN-based methods, including Message Passing GNN, Graph Convolutional Network (GCN), and Graph Attention Network (GAT). Fuzzy degree, betweenness, and closeness centralities are subsequently employed to incorporate weighted relational information, while a Message Passing FGNN is implemented for community detection. The framework is examined using a demo graph and real-world networks, including the Dolphin Social Network, Zachary's Karate Club Network, and Bitcoin Alpha Trust Network. The Bitcoin network provides weighted trust relationships and therefore enables detailed fuzzy network analysis. GAT further distinguishes influential relationships through learnable attention mechanisms. The comparative analysis uses Spearman's rank correlation coefficient for centrality measures and Silhouette scores for GNN-based community detection. Results demonstrate that fuzzy approaches outperform corresponding traditional methods, indicating that incorporating varying levels of trust and relational importance improves network representation and community analysis. The FGNN analysis of the Bitcoin Alpha Trust Network identifies ten communities, demonstrating its capability to capture complex relational structures. The proposed framework therefore provides a mathematical and computational foundation for uncertainty-aware social network analysis. Future research can extend fuzzy modeling to signed trust-distrust networks and advanced architectures such as fuzzy GAT, supporting more interpretable, reliable, and intelligent graph-based systems.
3	**Impact on SDG#__"** (min 500 words) (Please mention the impact of the relevant attained SDG)

SDG 9: Industry, Innovation and Infrastructure

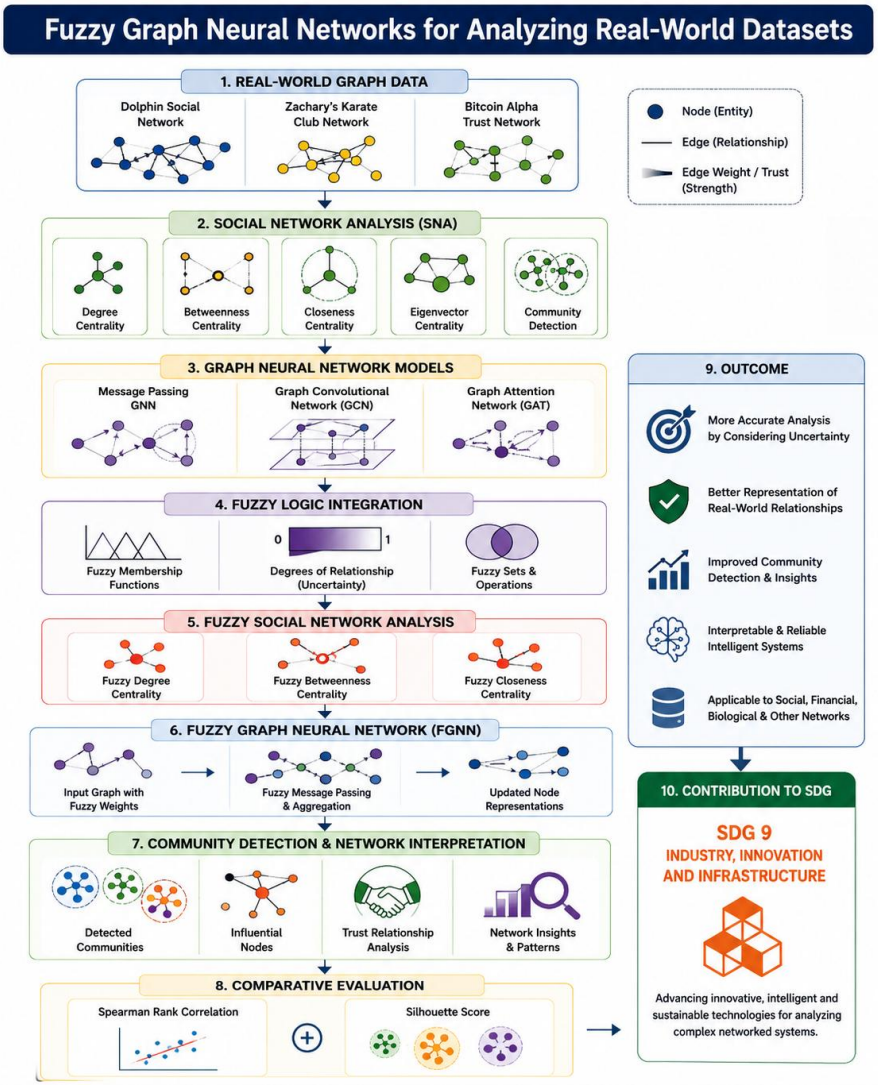
The proposed research contributes to **Sustainable Development Goal 9 (SDG 9): Industry, Innovation and Infrastructure** by advancing intelligent computational methods for analyzing complex and uncertain network-structured data. Modern digital systems generate increasingly interconnected information in the form of social networks, financial networks, communication systems, biological networks, recommendation systems, and other graph-based structures. Developing reliable and intelligent analytical techniques for such data is an important component of technological innovation and sustainable digital infrastructure.

This research contributes to this objective by integrating **fuzzy logic with Graph Neural Networks and Social Network Analysis**. Conventional graph methods generally represent relationships through fixed or crisp connections, whereas many real-world relationships involve different degrees of strength, trust, similarity, or uncertainty. The thesis addresses this limitation by introducing fuzzy membership concepts into network analysis and implementing fuzzy centrality measures and a Fuzzy Graph Neural Network for community detection.

The study further supports innovation by establishing a mathematical connection among **graph theory, fuzzy mathematics, and deep learning**. Its objectives explicitly include examining the mathematical framework of GNNs, synthesizing fuzzy logic, graph theory, and deep learning to establish FGNN foundations, and implementing these architectures on real-world datasets. This interdisciplinary approach can contribute to the development of more reliable AI systems capable of operating in environments where relationships cannot be adequately described using binary information.

The practical significance of the research is demonstrated through real-world network datasets, particularly the **Bitcoin Alpha Trust Network**, where weighted edges represent different levels of trust. Fuzzy analysis allows these varying levels of relational importance to be incorporated rather than treating every connection equally. The thesis reports that fuzzy methods provide analytical advantages over traditional approaches and that the FGNN successfully identifies community structures within the Bitcoin trust network.

Therefore, the research supports **SDG 9** by contributing to innovation in artificial intelligence, mathematical modeling, network science, and intelligent data analytics. Its broader impact lies in developing more uncertainty-aware, interpretable, and reliable computational frameworks that can potentially be extended to financial, social, healthcare, communication, and other network-based applications. Future extensions involving signed trust-distrust networks and advanced fuzzy GNN architectures can further strengthen the framework and expand its applicability to complex real-world digital systems.

4	**Scholar Name** (along with contact details) Urooj Naz roojnaz2000@gmail.com

5	**Supervisor & Co-supervisor Name** (along with contact details) Dr. Ubaida Fatima
6	**Graphical Abstract**